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EFFECT OF FLUID VISCOSITY ON THE DECAY OF SMALL DISTORTIONS OF A GAS BUBBLE FROM A SPHERICAL SHAPE

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The region of application of approximate methods for describing the effect of viscosity on the decay of small distortions of a gas bubble from a spherical shape is refined by comparing solutions obtained using the approximate methods and the exact Prosperittimodel. Approximate methods corresponding to a number of limiting cases are considered. The features of the errors arising in descriptions of the evolution of the distortions using approximate methods are found in the case of a significant effect of rotational fluid flow. A new approximate method is proposed.

Key words: *bubble dynamics, oscillation decay, distortion of the spherical shape.*

Introduction. Until recently, in problems of bubble dynamics with small distortions from a spherical shape, the effect of fluid viscosity in relation to its effect within the framework of the Navier–Stokes equations has been taken into account approximately. In particular, this has been implemented using the solution of the problem of decay of distortions in a spherical symmetric field of mass forces [1, 2]. A similar description of the viscosity effect is obtained under the assumption that the effect of fluid flow vorticity is manifested only in the surface layer [3]. Approximate methods for describing the viscosity effect are not always applicable. Their use is problematic, for example, in studies of single bubble sonoluminescence (SBSL) [4]. The SBSL phenomenon is the periodic emission of short light pulses by a gas bubble which performs radial oscillations in the current antinode of an ultrasonic standing pressure wave [4]. The discovery of the SBSL phenomenon in 1990 has stimulated research of distortions of micron-size bubbles from a spherical shape. The smaller the bubble size, the greater the viscosity effect. Therefore, in studies of SBSL, use has been made of the so-called exact method of accounting for viscosity [5] and a number of approximate method based on it [6, 7]. The method of accounting for viscosity in accordance with [5] is called exact in the sense that in the case of small distortions, it is equivalent to allowing for viscosity within the framework of the Navier–Stokes equations.

It is of interest to study whether the exact method of accounting for viscosity [5] can be replaced by an approximate method because the exact method is mathematical much more complicated than approximate methods [1–3, 6, 7] although it is much simpler than accounting for viscosity within the framework of the Navier–Stokes equations.

The exact method of accounting for viscosity [5] was used to study the decay of oscillations of the bubble shape. Thus, Asaki and Marston [8] studied the decay of nonspherical oscillations of bubbles which were acoustically trapped in fresh water and sea water. Roberts and Wu [9] give an asymptotic solution for the magnitude of distortion for large values of the free decay time. A number of asymptotic solutions were obtained in [10] using the Laplace transformation.

The goal of the present paper is to refine the region of application of the approximate methods of describing viscosity by comparing solutions obtained by approximate methods and for the exact model [5].

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1. Formulation of the Problem. At the time $t = 0$, a bubble with a small deviation of the shape from sphericity is in rest. Under the action of surface tension forces, the shape of the bubble begins to oscillate with an amplitude decay with time due to friction forces. In spherical coordinates r, θ, and φ, the equation of the surface is written as

$$F(r,\theta,\varphi,t) = r - R(1 + a_{ij}(t)Y_i^j(\theta,\varphi)) = 0.$$

Here R is the constant radius of the spherical component of the bubble shape, Y_i^j is a spherical harmonic of degree i and order j ($-i \leqslant j \leqslant i$). The parameter a_{ij} characterizes the dimensionless (referred to the radius R) deviation of the surface from a sphere in the form Y_i^j, whose amplitude is determined by its modulus $|a_{ij}(t)|$ and whose direction is determined by the sign of the product $a_{ij}Y_i^j$ (it is directed outward for $a_{ij}Y_i^j > 0$ and inward for $a_{ij}Y_i^j < 0$). The deviation a_{ij} is considered small ($|a_{ij}| \ll 1$). The bubble gas is considered inviscid, and its density is much smaller than the density of the fluid ρ_0. The equation for a_{ij} is written as [5]

$$\ddot{a}_{ij} + 2(i+1)(i+2)\beta_i\dot{a}_{ij} + (i+1)^2(i+2)^2a_{ij} + i(i+1)\beta_i(T_{ij}(1,\tau) - 2(i+1)\alpha_{ij}) = 0, \tag{1.1}$$

where

$$\beta_i = \frac{(i+1)(i+2)\nu}{\omega_i R^2}, \quad \omega_i^2 = \frac{\sigma}{\rho_0 R^3}(i^2-1)(i+2); \qquad \alpha_{ij} = \int_1^\infty T_{ij}(\xi,\tau)\,\frac{d\xi}{\xi^i},$$

$\xi = r/R$, $\tau = t\omega_i^*$, $\omega_i^* = \omega_i(i+1)^{-1}(i+2)^{-1}$, σ is the surface tension coefficient, $\nu = \mu/\rho_0$, μ is the dynamic viscosity, and β_i is an analog of the Reynolds number [11], in which the viscosity effect is characterized by the parameter $(i+1)(i+2)\nu$ and the characteristic velocity $\omega_i\boldsymbol{R}$ is the maximum (in θ) radial velocity of the fluid $\boldsymbol{u}$ on the bubble surface in the case where the oscillation frequency is $a_{ij} = 1$. It is assumed that $\nabla \times \boldsymbol{u}|_{\tau=0} = 0$; as a result, the rotational motion is toroidal and the function $T_{ij}(\xi,\tau)$ is defined by the expression $\nabla\times\boldsymbol{u} = \nabla\times T_{ij}R\omega_i^*Y_i^j\boldsymbol{e}_r$, where $\boldsymbol{e}_r$ is the unit vector of the axis r. The function $T_{ij}(\xi,\tau)$ is obtained from the equation

$$\frac{\partial T_{ij}}{\partial\tau} + \beta_i\Big(\frac{i(i+1)T_{ij}}{\xi^2} - \frac{\partial^2 T_{ij}}{\partial\xi^2}\Big) = 0 \tag{1.2}$$

subject to the boundary conditions

$$T_{ij}(1,\tau) = 2[(i+2)/(i+1)]\dot{a}_{ij} - 2\alpha_{ij}, \qquad T_{ij}(\infty,\tau) = 0. \tag{1.3}$$

At $t = 0$ we set $\dot{a}_{ij}(0) = 0$, $T_{ij}(\xi,0) = 0$, and $a_{ij}(0) = a_{ij}^0$.

The function $T_{ij}(\xi,\tau)$ is of importance in the analysis of the rotational component of the fluid flow. First, it determines the vorticity $\nabla\times\boldsymbol{u} = \boldsymbol{\omega} = \Omega_\theta\boldsymbol{e}_\theta + \Omega_\varphi\boldsymbol{e}_\varphi$, where $\Omega_\theta = (T_{ij}/(\xi\sin\theta))\,\partial Y_i^j/\partial\varphi$, $\Omega_\varphi = (T_{ij}/\xi)\,\partial Y_i^j/\partial\theta$, and $\boldsymbol{e}_\theta$ and $\boldsymbol{e}_\varphi$ are the unit vectors of the θ and φ axes. Second, the effect of the rotational component of the fluid flow is described by two terms of Eqs. (1.1) that depend on the function $T_{ij}(\xi,\tau)$.

Equations (1.1)–(1.3) are obtained from the Navier–Stokes equations under the assumption that the distortions of the bubble from a spherical shape are small [5]. The solution of system (1.1)–(1.3) is found numerically, and below it is called exact.

2. Approximate Methods of Accounting for Viscosity. The spatial distribution of the vorticity (and, hence, the quantity α_{ij}) ignoring the nonstationary nature of its diffusion is determined by its boundary value. In this case, $T_{ij}(1,\tau)$ depends on $\dot{a}_{ij}$, as follows from (1.3). As a result, the effect of rotational fluid flow in (1.1) degenerates into a certain correction of the coefficient at $\dot{a}_{ij}$ and for all approximate methods of describing the viscosity effect ignoring the nonstationary diffusion of the fluid flow vorticity, the equation for a_{ij} can be written as

$$\ddot{a}_{ij} + 2(i+1)(i+2)\beta_i(1+C_i)\dot{a}_{ij} + (i+1)^2(i+2)^2a_{ij} = 0, \tag{2.1}$$

where a particular expression of C_i corresponds to each method.

The solution of Eq. (2.1) for $a_{ij}(0) = a_{ij}^0$ and $\dot{a}_{ij}(0) = 0$ has the form

$$a_{ij} = a_{ij}^0\,\mathrm{e}^{-(i+1)(i+2)\Delta_i\tau}\cos\Big(\tau(i+1)(i+2)\sqrt{1-\Delta_i^2} - \psi_0\Big)/\sqrt{1-\Delta_i^2} \quad \text{for} \quad \Delta_i < 1, \tag{2.2}$$

$$a_{ij} = a_{ij}^0\,\mathrm{e}^{-(i+1)(i+2)\Delta_i\tau}\Big(K_1\,\mathrm{e}^{\tau(i+1)(i+2)\sqrt{\Delta_i^2-1}} + K_2\,\mathrm{e}^{-\tau(i+1)(i+2)\sqrt{\Delta_i^2-1}}\Big) \quad \text{for} \quad \Delta_i > 1, \tag{2.3}$$

$$\psi_0 = \arccos\sqrt{1-\Delta_i^2}, \quad K_1 = \frac{\Delta_i + \sqrt{\Delta_i^2 - 1}}{2\sqrt{\Delta_i^2-1}}, \quad K_2 = -\frac{\Delta_i - \sqrt{\Delta_i^2 - 1}}{2\sqrt{\Delta_i^2-1}}, \quad \Delta_i = (1+C_i)\beta_i.$$

Expression (2.2) describes the regime of decaying oscillations of the deviation, and expression (2.3) describes the regime of its decrease without oscillations. The deviation decreases at the maximum rate without oscillations for $\Delta_i = 1$.

In the present study, we use the following approximate methods.

Method I [1–3]. It is assumed that $\partial T_{ij}/\partial\tau = 0$ for $\xi > 1$, which follows from (1.2) as $\beta_i \to 0$. In this case, if $T_{ij}(\xi, 0) = 0$ ($\xi > 1$), then $T_{ij}(\xi,\tau) = 0$ for $\tau > 0$. This implies that the energy losses in the formation of vorticity on the bubble surface are taken into account and the opposite effect of rotational fluid flow on the bubble surface is ignored. In (1.1), the energy losses are taken into account by the term dependent on $T_{ij}(1,\tau)$. For a fixed i, the accuracy of this method increases with decrease in ν and/or with increase in R since under these conditions the fluid vorticity outside the bubble surface becomes less considerable. In the case of fixed ν and R, this method can be used only for small i. Method I corresponds to $C_i = i(i+1)^{-1}$.

The solution of the problem of free decay of distortions using method I was employed to describe viscosity in a study [2] of high-frequency distortions of a radially oscillating bubble in a low-viscosity fluid. In this case, use was made of the condition $\sqrt{i(-R^3\ddot{R} + \sigma R i^2)} \gg 2\nu i^2 R^2/R_0^2$, where R and R_0 are the current and equilibrium radii of the bubble. For $R = \text{const}$ and $i \gg 1$, this condition is equivalent to the condition $\beta_i \ll (1 + C_i)^{-1}$.

Method II. This method uses the quasistatic solution of Eqs. (1.2): $T_{ij}(\xi) = T_{ij}(1)\xi^{-i}$, which is valid for $\beta_i \to \infty$, whence $C_i = -3i(i+1)^{-1}(2i+1)^{-1}$. The accuracy of this method increases for fixed i with increase in ν and/or with decrease in R, and for fixed ν and R with increase in i. Under these conditions, the rate of vorticity diffusion in the fluid becomes increasingly higher with respect to the rate of variation in its boundary value.

In [6], the effect of rotational fluid flow is evaluated by the expression $\alpha_{ij} = T_{ij}(1,\tau)\delta$. For the quasistatic solution, we have $T_{ij}(1,\tau) = 2T_{ij}(1+\delta,\tau)$. Then, $C_i = -i(i+1)^{-2}$. In [6], the linearization of this relation is used: $C_i = -2(i+1)^{-1}$. These expressions correspond neither to the case of small values of β_i (method I) nor to the case of its large values (method II).

Method III. This method is a particular case of method II as $i \to \infty$, so that $C_i = 0$. The accuracy of method III, as well as that of method II, increases with increase in i for fixed values of ν and R. The equality $C_i = 0$ corresponds to solution (1.1) for $T_{ij}(1,\tau) - 2(i+1)\alpha_{ij} = 0$. The last relation implies that the energy losses due to the formation of vorticity on the bubble surface are compensated for by the opposite effect of rotational fluid flow. For any i, these losses are not equal to zero since the coefficient C_i of method I is in the range of 2/3 to 1.

The difference between methods I–III can be explained as follows. In method I, rotational fluid flow is ignored and the energy losses due to the production of this flow are taken into account. The presence of vorticity diffusion would reduce these losses due to both a decrease in the vorticity at the interface [the second term in (1.3)] and due to the opposite effect of rotational flow on the variation in the deviation [the last term in (1.1)]. Because of the greater consumption of the fluid flow energy, the rate of variation in the deviation will be smaller in describing the viscosity effect by method I than by methods II and III, which take into account vorticity diffusion. In method III, the energy consumption is greater than that in method II. This indicates that in method II, the rotational fluid flow (here it is in essence a result of both the vorticity diffusion from the boundary of the bubble and shear motion in the viscous fluid) always has higher energy than the energy expended for the production of vorticity at the interface.

Method IV. In this method, transition is performed from the region of small values of β_i (method I) to the region of large values (method II). As in [6], use is made of the estimate $\alpha_{ij} = T_{ij}(1,\tau)\delta$. For small β_i, the dependence $\delta(\beta_i)$ is determined by solving the problem of decay of plane waves in a viscous fluid [11], and for large β_i, it is determined by the quasistatic solution of Eq. (1.2), so that

$$C_i = \frac{i[1 - 2(i+1)\delta]}{(i+1)(1+2\delta)}, \qquad \delta = \min\left(\sqrt{\frac{\beta_i}{(i+1)(i+2)}}, \frac{1}{2i-1}\right). \tag{2.4}$$

It is easy to see that as β_i tends to zero and infinity, method IV approaches methods I and II, respectively.

3. Decay for a Weak Effect of Viscosity. The features of the decay of nonspherical oscillations of the bubble for $i = 2$ and $-2 \leqslant j \leqslant 2$ for small values of the parameter β_2 are illustrated in Fig. 1 for the case $\beta_2 = 0.096$ (which corresponds, for example, to $\sigma = 0.073$ N/m, $R = 4.5 \cdot 10^{-6}$ m, $\rho_0 = 10^3$ kg/m^3, and $\nu = 0.5 \cdot 10^{-6}$ m^2/sec). Time dependences of the quantity $\log|a_{2j}/a_{2j}^0|$ obtained using the exact and approximate methods of accounting

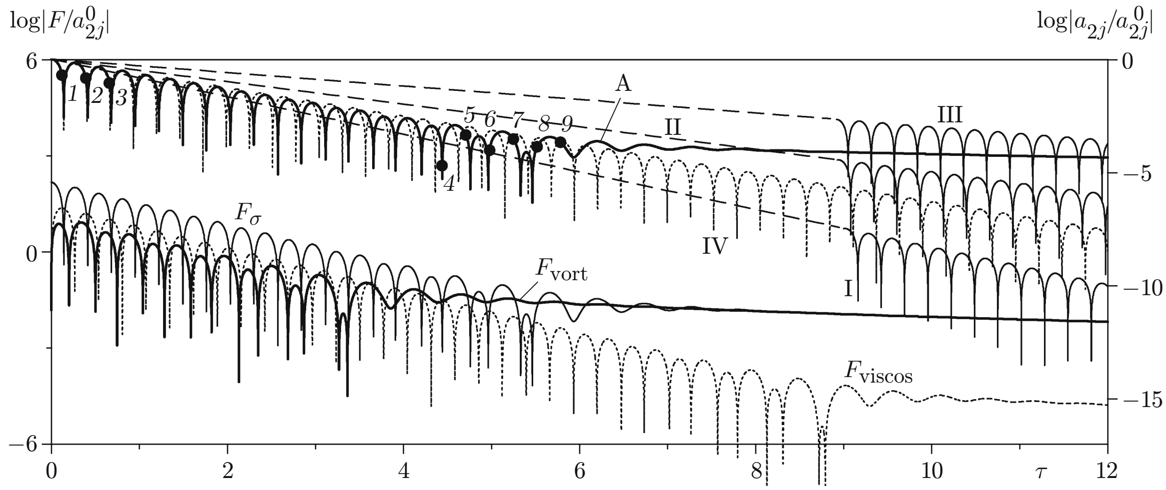

Fig. 1. Time dependence of the values of log $|a_{2j}/a_{2j}^0|$ (curve A and curves I–IV) and log $|F/a_{2j}^0|$ for $\beta_2 = 0.096$: curve A corresponds to the exact method of accounting for fluid viscosity, and curves I–IV correspond to approximate method numbers; the dashed segments of the curves show the variation in the oscillations frequency; points 1–9 denote a number of local maxima $|\dot{a}_{2j}|$.

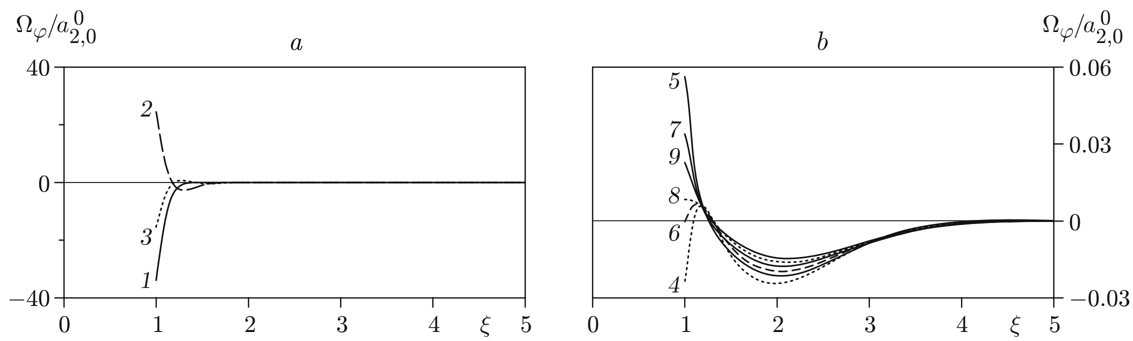

Fig. 2. Spatial distributions of the quantity $\Omega_\varphi/a_{2,0}^0$ along the line $\theta = \pi/4$ for $\beta_2 = 0.096$ for the times indicated in Fig. 1 by points 1–9.

for viscosity are plotted at the top of the figure, and at the bottom there are time dependences of log $|F/a_{2j}^0|$, where F denotes the forces representing the second (F_σ), the third ($F_{\rm viscos}$), and the fourth ($F_{\rm vort}$) terms in (1.1). In this case, F_σ characterizes the surface tension, $F_{\rm viscos}$ the effect of viscous friction (from irrotational fluid flow), which is proportional to the velocity of the interface, and $F_{\rm vort}$ the effect of rotational fluid flow.

If the viscosity is taken into account exactly, the quantity a_{2j} up to $\tau \approx 4$ oscillates with a slightly varying period and with an amplitude decreasing almost exponentially. The center of these oscillations is a slightly nonspherical shape (which is not evident in curve log $|a_{2j}/a_{2j}^0|$ but is apparent from the not purely exponential variation in the oscillation amplitude $F_{\rm vort}$). In the initial time interval, the bubble shape varies predominantly under the action of the forces F_σ and $F_{\rm viscos}$. The former force is markedly larger, which ensures the oscillation regime. Here the force $F_{\rm vort}$ is smaller than $F_{\rm viscos}$ and plays the role of a complement of it. For $\tau \approx 4$, as the oscillations the bubble shape are decaying, the force $F_{\rm vort}$ and F_σ become comparable. The bubble surface begins to perform oscillations relative to a pronounced (compared to the oscillations amplitude) nonspherical shape with a shift of the center of the oscillations of a_{2j} to the region of values of the sign opposite to the initial one. The oscillations of the surface proceeds first ($4 < \tau < 5.8$) with transition through a spherical state and then ($5.8 < \tau < 8$) without it. At $\tau > 8$, the deviation a_{2j} decreases without oscillations under the power law $\tau^{-(i+1/2)} = \tau^{-5/2}$ [9].

Figure 2 shows the evolution of the rotational fluid flow for $i = 2$ and $j = 0$, when $\Omega_\theta = 0$ (for $j \neq 0$, the picture is qualitatively the same). The figure gives distributions of $\Omega_\varphi/a_{2,0}^0$ along the line $\theta = \pi/4$ (here its values are maximum for θ) at times t_{1-9} corresponding to a number of local maxima of the rate of variation in $a_{2,0}$.

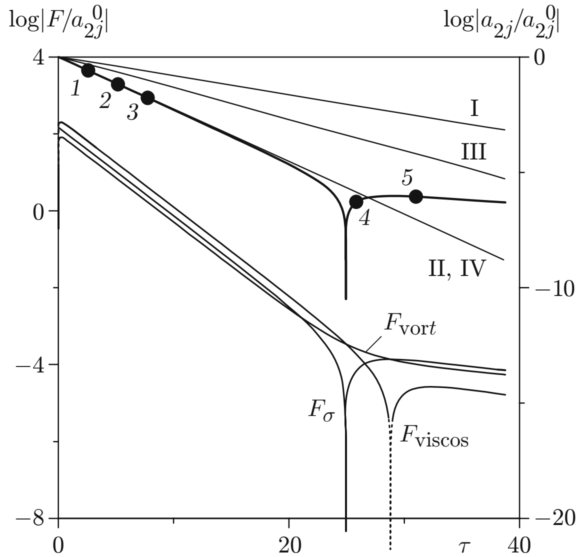

Fig. 3. Time dependences of log $|a_{2j}/a^0_{2j}|$ and log $|F/a^0_{2j}|$ for $\beta_2 = 19.1$ (notation the same as in Fig. 1).

According to Fig. 2a, before the time $\tau \approx 4$, the main perturbations in the vorticity field $\boldsymbol{\omega}$ occur near the bubble (in the near field of the fluid of length on the order of δ). In this period of time, they vary in an oscillation regime with a decreasing amplitude. Their evolution is determined here by relatively high velocities of motion of the bubble surface. The small phase shift in the variations of F_{vort} and F_{viscos} is due to a manifestation of the fluid inertia in the near field. With time (for $\tau > 4$ in Fig. 2b), the amplitude of perturbations of the vector $\boldsymbol{\omega}$ in the near field becomes comparable with the perturbation amplitude in a considerably more extended external region (far field). At $\tau > 4$, the oscillations of the bubble surface continue to influence only the variation in the vector $\boldsymbol{\omega}$ in the near field. The quantity $\boldsymbol{\omega}$ in the far field and its effect on the bubble surface vary during the entire process without oscillations and much more slowly than in the near field, namely under the law $\tau^{-5/2}$. The remote and most extended part of the far field is formed as a result of the first (before the time τ_1) and the fastest oscillations. Therefore, the sign of $\Omega_\varphi/a^0_{2,0}$ remains negative there throughout the process. The contribution of this part of the far field to the quantity F_{vort} results in displacement of the center of oscillations of the force F_{vort} and, hence, the center of oscillations of $a_{2,0}$ to the region of negative values. The latter varies under a power law, and the oscillations decay exponentially; therefore in a certain period of time, the maximum rate of variation in $a_{2,0}$ and the rate of variation in the center of the oscillations become almost equal. As a result, the shape oscillations cease and the deviation $a_{2,0}$ slowly approaches zero under a power law.

In the example considered, method IV provided the best approximation of the exact solution (see Fig. 1). For a successful choice of C_2, as noted above, the rate of decaying of oscillations of a_{2j} in the exact solution on the segment of exponential decay of these oscillations (up to $\tau \approx 5$) can be well approximated by solution (2.2). The period of decaying oscillations for the thus obtained approximate solution is always smaller than that for the exact solution, which is explained by a manifestation of the fluid inertia in its near field. With increase in β_2, the difference between the periods of decaying oscillations of the exact and approximate solutions increases.

4. Decay for a Strong Effect of Viscosity. The features of nonspherical oscillations of a bubble for $i = 2$ and $-2 \leqslant j \leqslant 2$ and large values of the parameter β_2 are illustrated in Fig. 3 for the case $\beta_2 = 19.1$ (which corresponds, for example, to $\sigma = 0.073$ N/m, $R = 4.5 \cdot 10^{-6}$ m, $\rho_0 = 10^3$ kg/m^3, and $\nu = 10^{-4}$ m^2/sec). In this case, the results of calculations using methods II and IV coincide.

In the case of exact allowance for viscosity, the deviation a_{2j} decreases almost exponentially up to $\tau \approx 15$. As in the case of small values of β_2 considered above, in the initial time interval, the fluid flow is affected primarily by the forces F_σ and F_{viscos}. However, here F_{viscos} is markedly larger than F_σ, because of which a_{2j} decreases exponentially and without oscillations. In this case, for $\tau < 0.1$, we have the relation $F_\sigma + F_{\text{vort}} > F_{\text{viscos}}$ (F_σ and F_{vort} are of the same sign), which initiates motion of the surface. Then, the indicated relation becomes

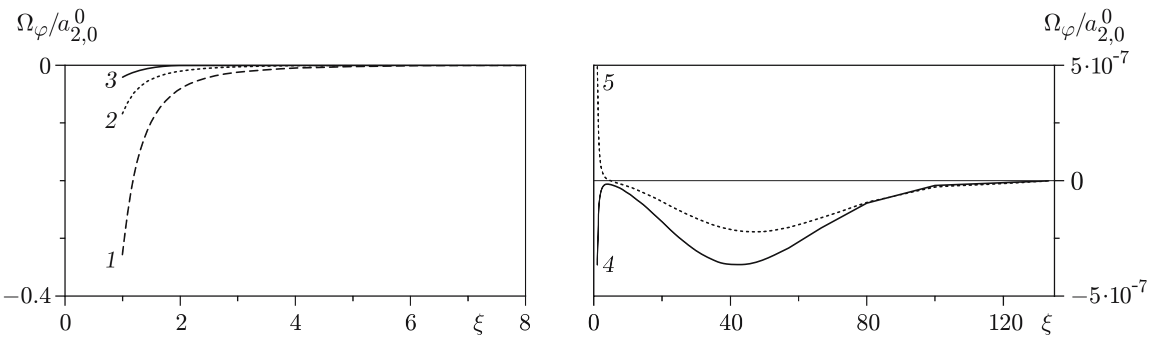

Fig. 4. Spatial distributions of the quantity $\Omega_\varphi/a^0_{2,0}$ along the line $\theta = \pi/4$ for $\beta_2 = 19.1$ for the times indicated in Fig. 3 by points 1–5.

opposite ($F_\sigma + F_{\rm vort} < F_{\rm viscos}$), which causes deceleration of the motion of the surface at $0.1 < \tau < 15$. In the initial time interval $0.1 < \tau < 15$, the value of the force $F_{\rm vort}$, as in the case of small values of the parameter β_2, is proportional to the rate of variation in a_{2j}, but here $F_{\rm vort}$ plays the role of a complement of F_σ.

The position of equilibrium of the deviation a_{2j} for the system of all acting forces is in the region of negative values of a_{2j}. This becomes appreciable for $\tau > 20$, when the decrease in $F_{\rm vort}$ slows down significantly with transition from the exponential law to a power law. The bubble shape passes through a spherical state, after which it approaches this state from the other side with a deviation decreasing according to a power law under the action of the force F_σ and its opposite force $F_{\rm vort}$.

The evolution of the rotational fluid flow for $\beta_2 = 19.1$ is illustrated in Fig. 4 for the case $i = 2$, $j = 0$, and $\theta = \pi/4$. Because of the strong effect of viscosity, the field of the vector $\boldsymbol{\omega}$ for $0.1 < \tau < 15$ is nearly quasistatic (curves 1–3 in the figure practically do not differ from the quasistatic solution). As for small β_2, in this case the motion of the surface influences the variation of $\boldsymbol{\omega}$ only in the near field. In the far field, the vector $\boldsymbol{\omega}$ varies with time much more slowly (according to a power law) than in the near field (curves 4 and 5 in Fig. 4). Subsequently, (for $\tau > 20$), the force effect of the vorticity in the far field on the bubble surface becomes predominant, resulting in a transition from the exponential law of variation of $F_{\rm vort}$ to a power law. Here the equilibrium position of the deviation $a_{2,0}$ is in the region of negative values for the same reason as for small β_2: upon a single oscillations of the surface throughout the process, the sign of $\Omega_\varphi/a^0_{2,0}$ in the far field remains negative.

The segment of exponential decrease in a_{2j} of the exact solution (up to $\tau \approx 20$) is well approximated by solution (2.3) with a successful choice of C_2, which is achieved in methods II and IV.

5. Estimation of the Approximate Methods of Accounting for Viscosity. A decrease in the viscosity effect (for $\beta_2 < 0.096$) in the exact solution leads to an increase in the time interval of decaying oscillations of a_{2j} with an amplitude decreasing almost exponentially. In this interval, method I provides increasingly good approximations of the decay rate and the oscillations period. With increase in the viscosity effect (for $\beta_2 > 19.1$) in the exact solution, the time interval increases as a_{2j} decreases almost exponentially. In this interval, method II gives a good approximation. Between $\beta_2 = 0.096$ and 19.1 there is a range of values of β_2, for which the above-mentioned regimes of variation in the deviation are not observed, which is due to the rapid manifestation of the vorticity effect in the far field.

To estimate the approximate methods of accounting for viscosity, we use the following criteria. For small values of β_2, the approximate method describes the viscosity effect well enough if the rates of decrease in the amplitudes of decaying oscillations of the deviation a_{2j} in the exact and approximate solutions remain close up the amplitudes decrease by two orders of magnitude. For large β_2, the approximate method describes the viscosity effect well enough if it satisfactorily approximates the decrease in the deviation up to the moment its value decreases by two orders of magnitude.

Figure 5a gives a number of curves of the coefficient C_2 versus the parameter β_2. The solid curve corresponds to the best (according to the above-mentioned criteria) approximation of the exact solution by the solution of Eq. (2.1) with an appropriate choice of the coefficient C_2 in it. In the region where a satisfactory approximation using these criteria was not achieved for any values of C_2 (in the range $0.11 < \beta_2 < 2.1$), a solid curve segment is

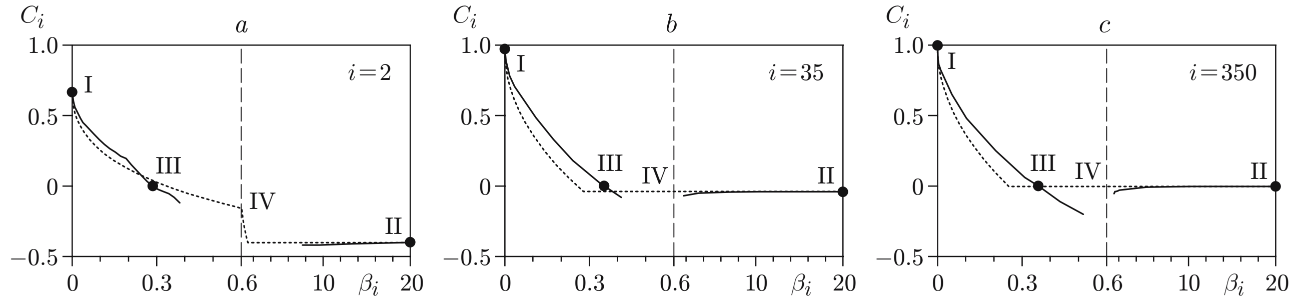

Fig. 5. Coefficient C_i versus the parameter β_i for the exact (solid curve) and approximate (dotted curve; method IV) methods of accounting for the viscosity of the fluid. For approximate methods I–III, the corresponding values of C_i are shown by points.

absent (to avoid overloading the figure, instead of the relations $C_i = \text{const}$ for approximate methods I–III, we give the points indicating the corresponding values of C_i only for those values of β_i for which these methods give the best approximation of the exact method). Among methods I–IV, method IV gives a more satisfactory approximation over a broad range of β_2.

Similar dependences of the coefficient C_i on β_i for $i = 35$ and 350 are given in Fig. 5b and c, respectively. It is evident that for large values of i, method IV also gives the best approximation over a broad range of β_i.

As i increases, the frequency and rate of decay of oscillations of a_{ij} increase for small β_i and the rate of almost exponential decrease in a_{ij} increases for large β_i. With increase in i, the effect of the far field of vorticity on the value of a_{ij} becomes appreciable for smaller values of a_{ij}; in this case, the region of small values of β_i, for which a_{ij} decreases in the regime of decaying oscillations, becomes increasingly larger. The last circumstance allows one to hope that with increase in i, the exact solution can be approximated by solution (2.2) over an increasingly broader region of small values of β_i. However, as a result of manifestation of the fluid inertia in its near field, the period of decaying oscillations of the exact solution becomes much larger than that of the approximate solution (2.2). In aggregate with the very fast decay of oscillations of a_{ij} (by more than two orders of magnitude for one period), this makes it impossible to satisfy the above criterion of approximating the exact solution by solution (2.2) in the regions of β_i in Fig. 5b and c, where solid curves are absent.

Conclusions. For the decay of nonspherical oscillations of a bubble in a viscous fluid, it is possible to distinguish two fields of fluid flow vorticity, which are different in the nature of effect on the bubble surface: near and far fields. The near field is located near the bubble in a layer of thickness on the order of δ [δ is defined by expression (2.4)], and the far field is outside this layer. The near field is determined by the motion of the interface. Its effect on the bubble surface is proportional to the rate of variation in its shape. The far field and its effect on the bubble surface are determined by the nonstationary nature of vorticity diffusion and practically do not depend on the motion of the surface. The near field first has a predominant effect, after which the effects of the near and far fields become comparable for some time, and then the far field begins to play a determining role. In the case of the predominant effect of the near field and low viscosity, the bubble surface varies in the form of exponentially decaying oscillations. If the viscosity is high, the deviation of the bubble from a spherical shape decreases almost exponentially without oscillations. In the case of the predominant effect of the far vorticity field for any viscosity, the deviation decreases according to a power law without oscillations. In this case, at each point, the deviation from a spherical shape is always opposite to the initial one. In the region where the effects of the far and near fields are comparable, the bubble surface for low viscosity varies in the form of decaying oscillations relative to the nonspherical shape first with transition through a spherical state and then without it. For high viscosity, the bubble surface varies without oscillations but with one transition through a spherical shape. The variation in the deviation can be satisfactorily described by the approximate methods of accounting for viscosity ignoring the effect of the far field of vorticity only in the cases of the predominant effect of the near field.

For not too low and not too high (moderate) viscosity, the approximate methods do not provide a satisfactory description of its effect because of the very fast manifestation of the effect of rotational fluid flow in the far field (for low-frequency harmonics) and/or because of the increasingly significant effect of the fluid inertia in its near field (for high-frequency harmonics).

Each of the approximate methods of accounting for viscosity is better than the others only in a narrow range of parameters of the problem. An approximate method is proposed which provides a satisfactory Description of the decrease in the deviation in a considerably broader range of parameters of the problem.

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